

## P.B. SIDDHARTHA COLLEGE OF ARTS & SCIENCE

Siddhartha Nagar, Vijayawada – 520 010 Reaccredited at 'A+' level by NAAC Autonomous & ISO 9001:2015 Certified

# Title of the Course: ORDINARY DIFFERENTIAL EQUATIONSSemester: I

Course Code	23MA1T2	Course Delivery Method	Blended Mode
Credits	5	CIA Marks	30
No. of Lecture Hours / Week	5	Semester End Exam Marks	70
Total Number of Lecture Hours	75	Total Marks	100
Year of Introduction : 2020-2021	Year of offering : 2023-2024	Year of Revision: 2023-24	Percentage of Revision :20%

**Course Objectives :** The main objective of this course is to learn various methods for finding solutions of an ordinary differential equation and to study the characteristics of solutions of differential equations.

Course Outcomes: After successful completion of this course, students will be able to

CO1: solve linear differential equations of second order. (PO3)

CO2: determine the power series solutions of differential equations. (PO1)

CO3: study the properties of Legendre and Bessel polynomials. (PO1)

CO4: solve the system of linear equations. (PO1)

CO5: understand the concept of existence and uniqueness of solutions. (PO5)

## UNIT-I

**Second order linear equations**: Introduction, The general solution of the homogeneous equation, The use of a known solution to find another, The homogeneous equation with constant coefficients, The method of undetermined coefficients, The method of variation of parameters. (Sections 14 to 19 of Chapter 3 of [1])

#### UNIT-II

**Power series solutions and special functions**: Introduction, A review of power series, Series solutions of first order equations, Second order Linear equations-Ordinary points, Regular singular points, Regular singular points(continued)

(Sections 26 to 30 of Chapter 5 of [1])

#### **UNIT-III**

**Some special functions of Mathematical Physics**: Legendre polynomials, Properties of Legendre Polynomials, Bessel functions, Properties of Bessel functions. (Sections 44 to 47 of chapter 8 of [1])

#### **UNIT-IV**

**Systems of Linear Differential Equations:** Introduction, Systems of first order equations, Model of arms competitions between two nations, Existence and uniqueness theorem, Fundamental Matrix, Non homogeneous linear systems, Linear systems with constant coefficients.[Sections 4.1 to 4.7 of Chapter 4 of Text Book (2)]

#### **UNIT-V**

**Existence and Uniqueness of solutions:** Introduction, Successive approximations, Picard's theorem. [Sections 5.1 to 5.4 of chapter 5 of Text Book(2)]

### **PRESCRIBED BOOKS :**

- G.F. Simmons, Differential equations with Applications and Historical Notes, Second Edition, Tata McGraw Hill, 2003.
- 2. S.G. Deo, V. Lakshmi kantham and V. Raghavendra "Text Book of Ordinary Differential Equations, Second edition, Tata McGraw Hill Pub., New Delhi, 1997.

#### **REFERENCE BOOKS :**

- 1. Earl.A. Coddington "An Introduction to Ordinary Differential Equations", PHI.
- D. Somasundaram, "Theory of Ordinary Differential Equations", Narosa Publications, 2001.

#### Course has Focus on : Foundation

#### Websites of Interest: 1. www. nptel.ac.in

- 2. www.epgp.inflibnet.ac.in
- 3. <u>www.ocw.mit.edu</u>

#### P B SIDDHARTHA COLLEGE OF ARTS AND SCIENCE::VIJAYAWADA (An autonomous college in the jurisdiction of Krishna University) M. Sc. Mathematics **First Semester ORDINARY DIFFERENTIAL EQUATIONS – 23MA1T2**

## Time: 3 hours

#### **SECTION-A**

## Answer all questions.

1 (a) By eliminating $c_1$ and $c_2$ , find the differential equation for the family of $c_1$	irves			
$\mathbf{y} = \mathbf{c}_1 \mathbf{e}^{\mathbf{x}} + \mathbf{c}_2 \mathbf{x} \mathbf{e}^{\mathbf{x}}$	(CO1, L3)			
(OR)				
(b) Solve the Euler's Equidimensional equation $x^2y^{11}+3xy^1+10y=0$	(CO1, L3)			
2 (a) Find the power series solution of $y^1 = 2xy$	(CO2, L2)			
(OR)				
(b) Find the power series expansion of the function $(1+x)^p$ , where p is a constant.				
	(CO2, L2)			
3 (a) Show that between any two zero's of $J_0(x)$ there is a zero of $J_1(x)$ . (OR)	(CO3, L3)			
(b) Show that $P_n(1) = 1$ and $P_n(-1) = (-1)^n$ .	(CO3, L3)			
4 (a) Define fundamental matrix of the system of linear differential equations and give an				
example. (OR)	(CO4, L1)			
(b) Explain the model for Arms competition between two nations.	(CO4, L1)			
(b) Explain the model for Arms competition between two hadons.	(CO4, L1)			
5 (a) State Lipschitz condition and give an example.	(CO5, L2)			
(OR)				
(b) Compute first two successive approximations of the equation $x' = x$ , $x(0) =$	1 (CO5, L2)			
SECTION – B				
Answer all questions. All questions carry equal marks.	(5X10=50)			
6 (a) If $v_1$ and $v_2$ are are two linearly independent solutions of $v^{11}+P(x)v^1+O(x)v=0$ on [a,b].				

6 (a) If  $y_1$  and  $y_2$  are are two linearly independent solutions of  $y^{11}+P(x)y^1+Q(x)y=0$  on [a,b], then show that  $c_1y_1 + c_2y_2$  is the general solution on [a,b]. (CO1, L3) (OR)

(b) Solve  $xy^{11} - (1+x)y^1 + y = x^2 e^{2x}$  using the method of variation of parameters. (CO1, L3)

Max. Marks: 70

(5x4=20)

7 (a) Find the general Solution of  $(1+x^2)y^{11}+2xy^1-2y = 0$  in terms of power series in x. (CO2, L3)

(OR)

- (b) Show that the equation  $4x^2y^{11}$   $8x^2y^1$ + $(4x^2+1)y=0$  has only one Frobenius Series Solution. Find the general solution. (CO2, L3)
- 8 (a) Derive Rodrigue's formula for Legendre polynomials. (CO3, L4)
  - (OR) (b) State and prove orthogonal property of Bessel polynomials. (CO3, L4)
- 9 (a) Find the fundamental matrix for x' = Ax where A =  $\begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}$  (CO4, L3)

(OR)  
(b) Determine 
$$e^{At}$$
 for the system x' = Ax where A =  $\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 3 \\ 0 & 1 & 0 \end{bmatrix}$  (CO4, L3)

10 (a) State and prove Picard's theorem.

(OR)

(CO5, L2)

(b) Find the first three successive approximations of the equation  $x^1 = e^x$ , x(0)=0. (CO5, L2)

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